

GCE

Mathematics

Unit **4724**: Core Mathematics 4

Advanced GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (OCR) Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	$\frac{2(1+x) + 3(3-x)}{(3-x)(1+x)}$ $\frac{11-x}{(3-x)(1+x)} \text{ oe isw}$	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>or $\frac{2(1+x)}{(3-x)(1+x)} + \frac{3(3-x)}{(3-x)(1+x)}$</p> <p>allow recovery from omission of brackets; brackets may be expanded in numerator</p> <p>numerator must be simplified B2 if unsupported</p> <p>denominator may be in expanded form at either stage eg $3 + 2x - x^2$</p>
1	(ii)	$\frac{(x+11)(x-3)}{(11+x)(11-x)} \text{ or } \frac{(x+11)(x-3)}{(121-x^2)}$ <p>their $\frac{11-x}{(3-x)(1+x)} \times$ their $\frac{(x+11)(x-3)}{(11+x)(11-x)}$</p> $\frac{-1}{(1+x)} \text{ oe cao}$	<p>M1*</p> <p>M1*dep</p> <p>A1</p> <p>[3]</p>	<p>allow $(x-11)(x+3)$ for numerator and / or $(x-11)(x+11)$ in denominator</p> <p>or $\frac{2}{(3-x)} \times$ their $\frac{(x+11)(x-3)}{(11+x)(11-x)}$ $+$ $\frac{3}{(1+x)} \times$ their $\frac{(x+11)(x-3)}{(11+x)(11-x)}$</p> <p>with at least one pair of their terms correctly cancelled out, allow if RH fraction only partially factorised</p>

Question		Answer	Marks	Guidance	
2	(i)	<p>$k(4\mathbf{i} + 8\mathbf{j} - 8\mathbf{k})$ oe seen</p> <p>$[\overrightarrow{CP} =] -2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ or $[\overrightarrow{PC} =] 2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$</p> <p>their \overrightarrow{CP}. their $k(4\mathbf{i} + 8\mathbf{j} - 8\mathbf{k})$ evaluated</p> <p>$\overrightarrow{AB} \cdot \overrightarrow{CP} = 4 \times (-2) + 2 \times 8 + 1 \times (-8) = 0$ oe</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>allow eg (4, 8, -8)</p> <p>allow eg (-2, 2, 1)</p> <p>allow one numerical error</p> <p>arithmetic must be shown and be consistent</p>	<p>NB</p> <p>$\overrightarrow{AB} = 4\mathbf{i} + 8\mathbf{j} - 8\mathbf{k}$ $\overrightarrow{AP} = 3\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ $\overrightarrow{BP} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$</p> <p>NB P is (4, 7, -3)</p> <p>if 0, allow B4 for fully correct solution using eg Pythagoras or trigonometry</p>
2	(ii)	<p>$\frac{1}{2} \times$ their $AB \times$ their CP oe</p> <p>18</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>or $\frac{1}{2}ab\sin C$ from triangle ABC with a, b and C correct</p> <p>$AB = \sqrt{4^2 + 8^2 + (-8)^2} = 12$ $CP = \sqrt{(-2)^2 + 2^2 + 1^2} = 3$ $AC = \sqrt{5^2 + 4^2 + (-7)^2} = \sqrt{90}$ $BC = \sqrt{1^2 + (-4)^2 + 1^2} = \sqrt{18}$ $AP = 9$ and $BP = 3$ $A = 18.4^\circ, B = 45^\circ$ and $C = 116.6^\circ$</p>	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx} = \pm k e^{2x} \cos x \pm e^{2x} \sin x$ $\frac{dy}{dx} = 2e^{2x} \cos x - e^{2x} \sin x \text{ oe}$ <p><i>their</i> $\frac{dy}{dx} = 0$</p> <p>$\tan x = 2$ or</p> $\cos x = (\pm) \frac{1}{\sqrt{5}} \text{ or } \sin x = (\pm) \frac{2}{\sqrt{5}}$ <p>$x = 1.11$ and -2.03 cao</p> <p>$y = 4.09$ and -0.00765 cao</p>	<p>M1*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[6]</p>	<p>k is any constant</p> <p>ignore omission of “$e^{2x} = 0$ has no solution”,</p> <p>(1.11, 4.09) and / or (-2.03, -0.00765)</p> <p>or A1 for each correct pair of co-ordinates: mark to benefit of candidate</p> <p>extra values within range incur a penalty of one mark; or any finite value for x obtained from $e^{2x} = 0$ incurs a penalty of one mark</p> <p>Product Rule</p> <p>or $\sqrt{5} \cos(x + \tan^{-1} \frac{1}{2}) = 0$</p> <p>if A0A0, SC1 for all 4 values to greater precision 1.107..., -2.034..., 4.094..., -0.0076457... (or -0.007646)</p> <p>NB $x = 1.107148718$ and -2.034443936 $y = 4.094229238$ and -0.007645738</p> <p>ignore extra values outside range</p>

Question		Answer	Marks	Guidance
4	(i)	$8^{2/3} = 4$ $(1 - \frac{9x}{8})^{2/3}$ seen $1 + \left(\frac{2}{3}\right)\left(\frac{\pm 9x}{k}\right) + \frac{1}{2!}\left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)\left(\frac{\pm 9x}{k}\right)^2$ where k is an integer greater than 1 $4 - 3x - \frac{9}{16}x^2$ or $4(1 - \frac{3}{4}x - \frac{9}{64}x^2)$ cao	B1 M1 M1 A1 [4]	may be embedded ignore extra terms or better
4	(ii)	$-\frac{8}{9} < x < \frac{8}{9}$ or $ x < \frac{8}{9}$ isw cao	B1 [1]	

Question	Answer	Marks	Guidance
5	$\frac{dt}{dx} = k(x+1)^{-\frac{1}{2}} \text{ or } \frac{dx}{dt} = 2t \text{ from } x = t^2 \pm 1 \text{ oe}$ $\int kte^{2t} dt$ $kt \times \frac{1}{2} e^{2t} \pm k \int \frac{1}{2} e^{2t} dt$ $te^{2t} - \int e^{2t} dt$ $te^{2t} - \frac{1}{2} e^{2t}$ $\sqrt{x+1} e^{2\sqrt{x+1}} - \frac{1}{2} e^{2\sqrt{x+1}} + c \quad \text{cao www}$	<p>M1</p> <p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[6]</p>	<p>or eg $kdt = \frac{dx}{\sqrt{x+1}}$ oe</p> <p>k is any non-zero constant</p> <p>may be implied by the next A1</p> <p>$+c$ may be seen in previous line only for A1</p>
6 (i)	$\frac{\sin x \times -\sin x - \cos x \times \cos x}{\sin^2 x}$ <p>may be implied by $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$</p> <p>eg</p> $= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} \text{ and completion to}$ $\frac{-1}{\sin^2 x} \quad \text{AG}$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>or $-\sin x \times \frac{1}{\sin x} + \cos x \times -(\sin x)^{-2} \times \cos x$ oe</p> <p>eg</p> $= \frac{-\sin^2 x}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} \text{ oe and completion to}$ $\frac{-1}{\sin^2 x}$ <p>allow sign errors only if M0, SC1 for just</p> $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$ <p>need to see at least two correct, constructive steps and statement of given answer for A1</p> <p>NB $\sin^2 x + \cos^2 x = 1$ seen may be a constructive intermediate step</p>

Question	Answer	Marks	Guidance	
6 (ii)	<p>$\cos 2x = 2\cos^2 x - 1$ substituted in numerator</p> <p>$\sin 2x = 2\sin x \cos x$ substituted in denominator</p> $\frac{\sqrt{2} \cos x}{2 \sin^2 x \cos x}$ <p>$F[x] = \pm k \frac{\cos x}{\sin x}$</p> $F\left[\frac{1}{4}\pi\right] - F\left[\frac{1}{6}\pi\right]$ <p>$= \frac{1}{2}(\sqrt{6} - \sqrt{2})$ www AG</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>[6]</p>	<p>or alternative form of double angle formula plus Pythagoras leading to no term in $\sin^2 x$ in numerator</p> <p>k must not be obtained from square rooting a negative number</p> <p>eg $\frac{-\cos \pi/4}{\sqrt{2} \times \sin \pi/4} - \frac{-\cos \pi/6}{\sqrt{2} \times \sin \pi/6}$</p>	<p>may be awarded if not seen as part of fraction</p> $\text{NB } \int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \frac{1}{\sqrt{2} \sin^2 x} dx$ <p>$\text{NB } -\frac{\cos x}{\sqrt{2} \sin x}$</p> <p>eg $\frac{-1/\sqrt{2}}{\sqrt{2} \times 1/\sqrt{2}} - \frac{-\sqrt{3}/2}{\sqrt{2} \times 1/2}$</p> <p>at least one correct intermediate step following substitution needed as well as statement of given result</p> <p>eg $-\frac{\sqrt{2}}{2}(1-\sqrt{3})$</p>

Question	Answer	Marks	Guidance	
7	<p>LHS is $k(x+y)(1+\frac{dy}{dx})$</p> <p>$k = 2$</p> <p>$2y\frac{dy}{dx}$ on RHS from differentiating y^2</p> <p>$y^2 + Kxy\frac{dy}{dx}$ on RHS</p> <p>obtains a value of y from eg $(1+y)^2 = 1 \times y^2$ oe</p> <p>substitution of $x = 1$ and their y dependent on at least two correct terms seen following differentiation, even if follows subsequent incorrect manipulation</p> <p>$\frac{dy}{dx} = -\frac{3}{8}$ oe cao</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[7]</p>	<p>or $2x + 2y\frac{dy}{dx} + ky + kx\frac{dy}{dx}$ k is any positive integer</p> <p>K is any positive integer</p> <p>allow even if follows incorrect manipulation</p> <p>may be implied by $1 + \frac{dy}{dx} = \frac{1}{4} - \frac{dy}{dx}$</p>	<p>some terms may appear on RHS with signs reversed</p> <p>if M0 in middle scheme, SC1 for three terms out of four completely correct with $k = 2$</p> <p>may appear on LHS with sign reversed</p> <p>NB $K = 2$; may appear on LHS with signs reversed</p> <p>NB $y = -0.5$</p> <p>or $\frac{dy}{dx} = \frac{2-1-0.25}{-1-2+1}$</p> <p>NB $\frac{dy}{dx} = \frac{2x+2y-y^2}{2xy-2x-2y}$</p> <p>-0.375</p>

Question		Answer	Marks	Guidance
8	(i)	$\frac{dP}{dt} = \frac{k}{P}$ $k = 100 \text{ from } \frac{dP}{dt} = \frac{k}{P}$ $\int PdP = \int (\text{their } k) dt$ $\frac{P^2}{2} = kt + c$ <p>substitution of $t = 0$ and $P = 100$</p> $P = \sqrt{10000 + 200t} \text{ or } 10\sqrt{100 + 2t}$ <p>or $P = \sqrt{200(50 + t)}$ isw cao</p>	<p>B1</p> <p>or $\frac{dP}{dt} = \frac{1}{kP}$</p> <p>B1</p> <p>or $k = 0.01$ from $\frac{dP}{dt} = \frac{1}{kP}$</p> <p>M1*</p> <p>allow $k = 1$</p> <p>A1</p> <p>or $t = \frac{P^2}{2k} + d$</p> <p>M1dep*</p> <p>may follow incorrect algebraic manipulation, but equation must include c (or d)</p> <p>A1</p> <p>[6]</p>	<p>k should be unspecified at this stage</p> <p>may be seen later</p> <p>allow omission of \int and recovery of omission of one operator for M1*A1</p> <p>if M0, SC2 for $\ln P = kt + c$ thereafter only M1 may be earned</p> <p>NB $c = 5000$ or $d = -50$</p> <p>allow recovery from eg use of x for P throughout, but withhold final A1 for eg $x = \sqrt{10000 + 200t}$</p>
8	(ii)	<p>$t = 8, P = 107.7$ or 108 so model was a good fit in 2008 oe</p> <p>$t = 13, P = 112(.2)$, so model was not appropriate in 2013 oe</p>	<p>B1</p> <p>or $t = 8.3(2)$ when $P = 108$ + comment</p> <p>B1</p> <p>or $t = 31.9(2)$ or 32 when $P = 128$ + comment</p> <p>comments may be in same sentence, but both values must be referenced</p> <p>[2]</p>	<p>value of P or t must be found and correct comment made in each case; comments may be in same sentence.</p> <p>if B0B0, SC1 for both values found no FT marks available</p> <p>comments on trends, extrapolation etc do not score</p> <p>just ticks / crosses etc do not score</p>

Question		Answer	Marks	Guidance
9	(i)	$\mu = 3$ soi $1 = 19 + (\text{their } 3) \times \alpha$ oe $[\alpha =] - 6$	B1 M1 A1 [3]	from $3 + 2\lambda = 4 + \mu$ and $5 + \lambda = 10 - \mu$ NB $\lambda = 2$ do not allow sign errors
9	(ii)	$2 \times 1 + 1 \times (-1) + 1 \times \alpha$ $\sqrt{(2^2 + 1^2 + 1^2)} \times \sqrt{(1^2 + (-1)^2 + \alpha^2)} \times \cos 60^\circ$ eg their $4 + 8\alpha + 4\alpha^2 = 6(2 + \alpha^2)$ $\alpha = 2$ cao	M1* M1* M1dep* A1 [4]	allow 1 sign error allow 1 slip, eg sign error or omission of power square both sides if M1M1M0, B2 for unsupported or alternative valid method NB $1 + \alpha = \sqrt{6} \times \sqrt{(2 + \alpha^2)} \times \cos 60^\circ$ NB $2\alpha^2 - 8\alpha + 8 = 0$
10	(i)	$\frac{A}{x} + \frac{B}{x+2}$ $x + 8 = A(x+2) + Bx$ soi $A = 4$ and $B = -3$	B1 M1 A1 [3]	award if only implied by answer clearing fractions successfully if M0, B1 for each value www

Question		Answer	Marks	Guidance
10	(ii)	<p>quotient (P) is 7</p> <p>$2x + 16$ seen</p> $7 + \frac{8}{x} - \frac{6}{x+2}$	<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p>	<p>if B0, B1 for $Q = 8$ and B1 for $R = -6$ www</p> <p>eg as remainder or in division chunking</p> <p>or allow $P = 7, Q = 8 R = -6$</p>
10	(iii)	<p>$t = f(x)$</p> $t = \frac{x}{x+2}$ <p>$y = 3 \times \text{their } \frac{x}{x+2} + \frac{4}{\text{their } \frac{x}{x+2}}$</p> <p>eg $\frac{3x^2 + (8+4x)(x+2)}{x(x+2)}$ and completion to</p> $y = \frac{7x^2 + 16x + 16}{x(x+2)} \text{ www AG}$	<p>M1*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>[4]</p>	<p>from $x = \frac{2t}{1-t}$;</p> <p>M0 for $t = g(y)$</p> <p>or B2 if unsupported</p> <p>at least one correct, constructive, intermediate step shown</p> <p>if M0M0, SC2 for substitution of $x = \frac{2t}{1-t}$ in RHS of given equation and completion with at least two correct, constructive intermediate steps to $y = 3t + \frac{4}{t}$ www</p>

Question		Answer	Marks	Guidance	
10	(iv)	$\int \text{their } \left(P + \frac{Q}{x} + \frac{R}{x+2}\right) [dx]$ $F[x] = 7x + 8 \ln x - 6 \ln(x+2)$ $F[2] - F[1]$ $7 - 4 \ln 2 + 6 \ln 3$	<p>M1*</p> <p>A1FT</p> <p>M1dep*</p> <p>A1</p> <p>[4]</p>	<p>where P, Q and R are constants obtained in (ii)</p> <p>allow recovery from omission of brackets in subsequent working</p>	<p>allow omission of dx</p> <p>if M0, SC1 for $Px + Q \ln x + R \ln(x+2)$ where constants are unspecified or arbitrary</p>

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